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Thermal entrance region for laminar forced convection in a circular tube with a power-law wall heat flux

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Abstract—Laminar forced convection in a circular tube is investigated with a boundary condition of prescribed axially varying wall heat flux, under the assumptions of hydrodynamically developed flow and of negligible axial conduction and viscous dissipation in the fluid. A condition on the asymptotic behaviour of the axial distribution of wall heat flux is established which is fulfilled by power-law varying axial distributions and which guarantees the existence of a thermally developed regime. It is proved that for all the axial distributions which fulfil this condition the asymptotic value reached by the local Nusselt number is 48/11, i.e. the same which holds for a uniform wall heat flux. For some power-law varying axial distributions of wall heat flux, a finite difference determination of the thermal entrance region is performed. In every numerical solution, the local Nusselt number tends asymptotically to 48/11.

INTRODUCTION

Laminar forced convection in circular ducts has been widely studied and important results obtained in this field have been reviewed in refs. [1, 2]. In most of these studies, the effects of axial conduction and of viscous dissipation in the fluid are neglected and various kinds of boundary condition at the wall of the tube are analysed. In particular, Sparrow and Patankar [3] show that the following boundary conditions:

(a) $q_w = \text{constant}$;
(b) $T_w = \text{constant}$;
(c) convective heat transfer to a fluid environment with a uniform temperature and a uniform heat transfer coefficient;

are particular cases of the boundary condition of exponentially varying wall heat flux, for a thermally developed regime. Indeed, a widely accepted result is that only axial distributions of wall heat flux which become exponential in the limit $x \to \infty$ are compatible with a thermally developed regime [1, 3–5]. Hasegawa and Fujita [5] have proved that the wall heat flux is exponential if the quantity $(T_w - T)/(T_w - T_b)$ is invariant along the axis of the tube. Obviously, the invariance of $(T_w - T)/(T_w - T_b)$ along the flow direction implies the invariance of the Nusselt number along this direction [6]. However, the existence of an asymptotically invariant radial distribution of dimensionless temperature is a condition much weaker than the existence of a radial distribution of dimensionless temperature which is exactly invariant along the axis of the tube for a finite length. Therefore, although the thesis stated in ref. [5] is correct, it does not imply that an exponential variation of the wall heat flux represents a necessary condition for the existence of an asymptotically invariant radial distribution of dimensionless temperature.

With the exception of studies on sinusoidal wall heat flux variation (see [7] and references therein), analyses of boundary conditions which cannot be reduced to that of exponential wall heat flux are very rare in the literature. For example, linearly varying axial distributions of wall heat flux have been studied by Hasegawa and Fujita [5]. However, only turbulent forced convection is analysed in ref. [5].

The aim of this paper is to prove that, for laminar forced convection in a circular duct with a fully developed velocity profile, every boundary condition of a prescribed wall heat flux $q_w(x)$ such that

$$\lim_{x \to +\infty} \frac{1}{q_w(x)} \frac{dq_w(x)}{dx} = 0 \tag{1}$$

yields a thermally developed regime, under the assumption that both axial conduction and viscous dissipation in the fluid are negligible. Moreover it is shown that, for the class of axial distributions of wall heat flux which fulfil equation (1), the value of the asymptotically invariant Nusselt number is 48/11, i.e. the same value which holds for a uniform wall heat flux. This result is illustrated by numerical analysis of heat transfer in the thermal entrance region for wall heat fluxes given by $q_w(x) = q_0(1+ax)^n$, for various values of n. Obviously, wall heat fluxes which can be expressed in the form $q_w(x) = q_0(1+ax)^n$ fulfil equation (1).

NOMENCLATURE

a	constant employed in equation (22) $[\mathrm{m}^{-1}]$
c_0	dimensionless constant employed in equation (16)
c_1	constant employed in equation (18) [m]
f	dimensionless function of r employed in equation (10)
f_b	bulk value of f
f_n	functions of r employed in equation (12) $[\mathrm{m}^{-n}]$
k	thermal conductivity $[\mathrm{W\ m^{-1}\ K^{-1}}]$
L	length of the tube [m]
n	dimensionless constant employed in equation (22)
N_r	number of intervals which define the mesh in direction r
N_x	number of intervals which define the mesh in direction x
Nu	Nusselt number, defined in equation (4)
Nu_∞	asymptotic Nusselt number
Pe	Peclet number, $Pe = 2r_0\bar{u}/\alpha$
q_w	wall heat flux $[\mathrm{W\ m^{-2}}]$
q_0	wall heat flux at $x = 0$ $[\mathrm{W\ m^{-2}}]$
r	radial coordinate [m]
r_0	radius of the tube [m]
r_j	radial coordinate at grid position (N_x, j) [m]
Δr	length of the intervals which define the mesh in direction r [m]
T	temperature [K]
T_b	bulk temperature [K]
$T_{\mathrm{b}\infty}$	asymptotic value of the bulk temperature [K]
$T_{N_x,j}$	temperature at grid position (N_x, j) [K]
T_0, $T_0^{(1)}$, $T_0^{(2)}$	inlet temperatures [K]
$T^{(1)}$, $T^{(2)}$	solutions of equation (5) with $T_0 = T_0^{(1)}$ and $T_0 = T_0^{(2)}$, respectively [K]
$\tilde{T}$	$= T^{(1)} - T^{(2)}$ [K]
u	axial component of the velocity $[\mathrm{m\ s^{-1}}]$
$\bar{u}$	mean value of u $[\mathrm{m\ s^{-1}}]$
x	axial coordinate [m]
Δx_0, Δx_i	lengths of the intervals which define the mesh in direction x [m].

Greek symbols

α	thermal diffusivity $[\mathrm{m^2\ s^{-1}}]$
γ	dimensionless constant employed in the definition of thermal entry length
ε	constant employed in equation (9) $[\mathrm{m}^{-1}]$
η	dimensionless radial coordinate, $\eta = r/r_0$
ϑ	dimensionless temperature defined in equation (24)
ϑ_b	bulk value of the dimensionless temperature
Λ	dimensionless parameter, $\Lambda = 2r_0 Pe\, a$
ξ	dimensionless axial coordinate, $\xi = x/(2r_0 Pe)$.

ASYMPTOTIC BEHAVIOUR OF THE TEMPERATURE FIELD

In this section, a laminar forced convection with a prescribed axial distribution of wall heat flux is considered, under the assumption that both axial conduction and viscous dissipation in the fluid are negligible. It is shown that the asymptotic value of the Nusselt number (i.e. its value for $x \to +\infty$) is independent of the radial distribution of the inlet temperature. Then, the asymptotic value of the Nusselt number is determined for a class of distributions of wall heat flux.

Let us consider a circular tube crossed by a Newtonian fluid whose flow is steady, laminar, incompressible and fully developed. An axial distribution of wall heat flux is prescribed. If forced convection is considered, the fluid velocity is directed along the axis and its magnitude is given by the well known Pouiseuille formula

$$u(r) = 2\bar{u}\left(1 - \frac{r^2}{r_0^2}\right). \tag{2}$$

The bulk temperature is given by

$$T_\mathrm{b}(x) = \frac{2}{\bar{u}r_0^2}\int_0^{r_0} T(x,r)u(r)r\,\mathrm{d}r \tag{3}$$

while the local Nusselt number can be expressed as

$$Nu = \frac{2r_0 q_\mathrm{w}(x)}{k[T(x,r_0) - T_\mathrm{b}(x)]}. \tag{4}$$

If both axial conduction and viscous dissipation in the fluid are negligible, the boundary value problem for the temperature distribution can be written as

$$\begin{cases} \dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right) = \dfrac{u(r)r}{\alpha}\dfrac{\partial T}{\partial x} \\ T(0,r) = T_0(r) \\ k\left.\dfrac{\partial T}{\partial r}\right|_{r=r_0} = q_\mathrm{w}(x). \end{cases} \tag{5}$$

Let us consider two boundary value problems with the same axial distribution of wall heat flux, $q_\mathrm{w}(x)$, but with two different radial distributions of the inlet temperature, $T_0^{(1)}(r)$ and $T_0^{(2)}(r)$. Let $T^{(1)}(x,r)$ and $T^{(2)}(x,r)$ be the solutions of equation (5) with

$T_0(r) = T_0^{(1)}(r)$ and $T_0(r) = T_0^{(2)}(r)$, respectively. Then, on account of equation (5), function $\tilde{T}(x,r) = T^{(1)}(x,r) - T^{(2)}(x,r)$ is a solution of the boundary value problem

$$\begin{cases} \dfrac{\partial}{\partial r}\left(r\dfrac{\partial \tilde{T}}{\partial r}\right) = \dfrac{u(r)r}{\alpha}\dfrac{\partial \tilde{T}}{\partial x} \\ \tilde{T}(0,r) = T_0^{(1)}(r) - T_0^{(2)}(r) \\ k\dfrac{\partial \tilde{T}}{\partial r}\Big|_{r=r_0} = 0. \end{cases} \tag{6}$$

Equation (6) shows that $\tilde{T}(x,r)$ can be interpreted as the fluid temperature field which develops within a tube with an adiabatic wall and a non-uniform inlet temperature $T_0^{(1)}(r) - T_0^{(2)}(r)$. It is well known that the temperature field tends to become uniform at sections sufficiently distant from $x = 0$ if the wall is adiabatic and no heat generation occurs within the fluid, for any radial distribution of the inlet temperature. In other words, the limit for $x \to +\infty$ of $\tilde{T}(x,r)$ is a constant. Therefore, for high values of x, the temperature fields $T^{(1)}(x,r)$ and $T^{(2)}(x,r)$ differ by a constant. Equations (3) and (4) show that only temperature differences are employed in the evaluation of the local Nusselt number. Therefore, in the limit $x \to +\infty$, the local Nusselt number evaluated with the temperature field $T^{(1)}(x,r)$ equals the Nusselt number evaluated with $T^{(2)}(x,r)$.

To summarize, the asymptotic behaviour of the Nusselt number can be determined independently of the boundary condition at $x = 0$. If one considers equation (5) and omits the boundary condition at $x = 0$, the reduced boundary value problem is given by

$$\begin{cases} \dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right) = \dfrac{u(r)r}{\alpha}\dfrac{\partial T}{\partial x} \\ k\dfrac{\partial T}{\partial r}\Big|_{r=r_0} = q_w(x). \end{cases} \tag{7}$$

Any solution of equation (7) can be employed in equation (4) to determine the asymptotic behaviour of the Nusselt number.

In the following, the evaluation of the asymptotic Nusselt number is performed for the axial distributions of wall heat flux which fulfil the condition

$$\lim_{x\to+\infty} \frac{1}{q_w(x)}\frac{\mathrm{d}q_w(x)}{\mathrm{d}x} = 0. \tag{8}$$

To perform this evaluation, let us consider the boundary value problem

$$\begin{cases} \dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right) = \dfrac{u(r)r}{\alpha}\dfrac{\partial T}{\partial x} \\ k\dfrac{\partial T}{\partial r}\Big|_{r=r_0} = q_w(x)\,\mathrm{e}^{\varepsilon x} \end{cases} \tag{9}$$

where $q_w(x)$ satisfies equation (8). Obviously, at the end of the calculation the limit $\varepsilon \to 0$ will be taken. A solution of equation (9) which holds for very large values of x has the form

$$T(x,r) = f(r)q_w(x)\frac{\mathrm{e}^{\varepsilon x}}{k\varepsilon}. \tag{10}$$

In fact, if $x \to +\infty$, by substituting equation (10) in equation (9) and by employing equations (2) and (8) one obtains

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}r}\left(r\dfrac{\mathrm{d}f}{\mathrm{d}r}\right) = \dfrac{2\bar{u}\varepsilon}{\alpha}r\left(1-\dfrac{r^2}{r_0^2}\right)f \\ \dfrac{\mathrm{d}f}{\mathrm{d}r}\Big|_{r=r_0} = \varepsilon. \end{cases} \tag{11}$$

If $f(r)$ fulfils equation (11), then equation (10) is an asymptotic expression, for $x \to +\infty$, of the solutions of equation (9). For small values of ε, equation (11) can be solved by the following perturbation method. Function $f(r)$ can be expressed as a power series in ε, namely

$$f(r) = f_0(r) + f_1(r)\varepsilon + f_2(r)\varepsilon^2 + \ldots. \tag{12}$$

By substituting equation (12) into equation (11), one obtains a boundary value problem for every order in the parameter ε. The 0th order boundary value problem is

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}r}\left(r\dfrac{\mathrm{d}f_0}{\mathrm{d}r}\right) = 0 \\ \dfrac{\mathrm{d}f_0}{\mathrm{d}r}\Big|_{r=r_0} = 0 \end{cases} \tag{13}$$

while the 1st order boundary value problem is

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}r}\left(r\dfrac{\mathrm{d}f_1}{\mathrm{d}r}\right) = \dfrac{2\bar{u}}{\alpha}r\left(1-\dfrac{r^2}{r_0^2}\right)f_0 \\ \dfrac{\mathrm{d}f_1}{\mathrm{d}r}\Big|_{r=r_0} = 1. \end{cases} \tag{14}$$

By employing the same method, one obtains the following boundary value problem at any order $n > 1$:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}r}\left(r\dfrac{\mathrm{d}f_n}{\mathrm{d}r}\right) = \dfrac{2\bar{u}}{\alpha}r\left(1-\dfrac{r^2}{r_0^2}\right)f_{n-1} \\ \dfrac{\mathrm{d}f_n}{\mathrm{d}r}\Big|_{r=r_0} = 0. \end{cases} \tag{15}$$

On account of equation (13), f_0 is a constant, i.e. $f_0(r) = c_0$, so that equation (14) yields

$$c_0 = \frac{2\alpha}{\bar{u}r_0} \tag{16}$$

$$f_1(r) = \frac{r^2}{r_0}\left(1-\frac{r^2}{4r_0^2}\right)+c_1. \tag{17}$$

The integration constant c_1 can be determined by employing equation (15) for $n = 2$. In particular, one obtains

$$c_1 = -\frac{7}{24} r_0. \tag{18}$$

Therefore, at the first order in the parameter ε, function $f(r)$ is given by

$$f(r) = \frac{2\alpha}{\bar{u} r_0} + \left[\frac{r^2}{r_0}\left(1-\frac{r^2}{4r_0^2}\right) - \frac{7}{24} r_0\right]\varepsilon + \cdots. \tag{19}$$

On account of equations (2) and (19), the bulk value of $f(r)$ evaluated at the first order in the parameter ε is

$$f_b = \frac{2\alpha}{\bar{u} r_0} + \frac{2\varepsilon}{\bar{u} r_0^2}\int_0^{r_0}\left[\frac{r^2}{r_0}\left(1-\frac{r^2}{4r_0^2}\right) - \frac{7}{24} r_0\right] u(r) r \, dr + \cdots$$

$$= \frac{2\alpha}{\bar{u} r_0} + \cdots \tag{20}$$

because the integral which appears in the term of order ε is zero. By employing equations (10), (19) and (20), the limit $\varepsilon \to 0$ of the asymptotic value of the Nusselt number, Nu_∞, can be easily evaluated:

$$Nu_\infty = \lim_{\varepsilon\to 0} \frac{2 r_0 q_w(x)\, e^{\varepsilon x}}{k[T(x, r_0) - T_b(x)]} = \lim_{\varepsilon \to 0} \frac{2 r_0 \varepsilon}{f(r_0) - f_b}$$

$$= \frac{2 r_0}{r_0(1-\frac{1}{4}) - \frac{7}{24} r_0} = \frac{48}{11} \cong 4.3636. \tag{21}$$

Equation (21) shows that, for every axial distribution of the wall heat flux which fulfils equation (8) and for every prescribed radial distribution of the inlet temperature, the limit for $x \to +\infty$ of the local Nusselt number is 48/11. Therefore, this value represents the asymptotic value of Nu not only for uniform wall heat flux [1], but also for any other axial distribution of wall heat flux which fulfils equation (8). This result will be illustrated in the following sections by a numerical analysis of the hydrodynamically developed and thermally developing forced convection for some non-uniform axial distributions of wall heat flux which satisfy equation (8).

DIMENSIONAL ANALYSIS OF THE BOUNDARY VALUE PROBLEM

In this section, the boundary value problem which describes the laminar forced convection in a circular tube with a prescribed wall heat flux $q_w(x) = q_0(1+ax)^n$ is written in a dimensionless form.

Let us consider the forced convection problem described by equation (5), with an axial distribution of wall heat flux $q_w(x) = q_0(1+ax)^n$ and with a uniform radial distribution of inlet temperature $T(0, r) = T_0$. Then equation (5) can be rewritten as

$$\begin{cases} \dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right) = \dfrac{u(r) r}{\alpha}\dfrac{\partial T}{\partial x} \\ T(0, r) = T_0 \\ k\left.\dfrac{\partial T}{\partial r}\right|_{r=r_0} = q_0(1+ax)^n. \end{cases} \tag{22}$$

The axial distribution of wall heat flux $q_w(x) = q_0(1+ax)^n$ fulfils equation (8) for every value of q_0, a and n. The axial variation of q_w is illustrated in Fig. 1 for some values of n.

Let us introduce the dimensionless radius $\eta = r/r_0$, the Peclet number $Pe = 2r_0\bar{u}/\alpha$, the dimensionless axial coordinate $\xi = x/(2r_0 Pe)$ and the dimensionless parameter $\Lambda = 2r_0 Pe\, a$, so that equation (22) yields

$$\begin{cases} \dfrac{\partial}{\partial \eta}\left(\eta\dfrac{\partial T}{\partial \eta}\right) = \dfrac{\eta}{2}(1-\eta^2)\dfrac{\partial T}{\partial \xi} \\ T(0, \eta) = T_0 \\ \left.\dfrac{\partial T}{\partial \eta}\right|_{\eta=1} = \dfrac{q_0 r_0}{k}(1+\Lambda\xi)^n. \end{cases} \tag{23}$$

By substituting the dimensionless temperature

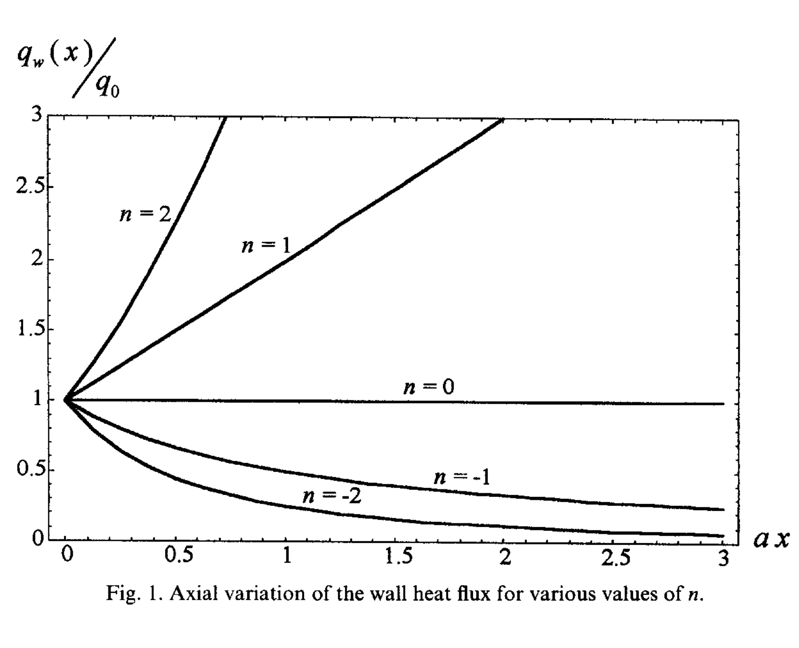

Fig. 1. Axial variation of the wall heat flux for various values of n.

$$\vartheta = k\frac{T-T_0}{q_0 r_0} \tag{24}$$

in equation (23), one obtains

$$\begin{cases} \dfrac{\partial}{\partial\eta}\left(\eta\dfrac{\partial\vartheta}{\partial\eta}\right) = \dfrac{\eta}{2}(1-\eta^2)\dfrac{\partial\vartheta}{\partial\xi} \\ \vartheta(0,\eta) = 0 \\ \left.\dfrac{\partial\vartheta}{\partial\eta}\right|_{\eta=1} = (1+\Lambda\xi)^n. \end{cases} \tag{25}$$

On account of equation (25), the dimensionless temperature distribution is a function of ξ and η which depends on the parameters Λ and n.

By employing equation (4), the local Nusselt number can be expressed as

$$Nu = \frac{2r_0 q_0(1+ax)^n}{k[T(x,r_0)-T_b(x)]}. \tag{26}$$

As a consequence of equations (3) and (24), the bulk value of the dimensionless temperature is given by

$$\vartheta_b(\xi) = k\frac{T_b(x)-T_0}{q_0 r_0}. \tag{27}$$

By employing equations (2), (3) and (24), $\vartheta_b(\xi)$ can also be expressed as

$$\vartheta_b(\xi) = 4\int_0^1 \vartheta(\xi,\eta)(1-\eta^2)\eta\,d\eta. \tag{28}$$

Equation (28) ensures that ϑ_b depends only on ξ, Λ and n. On account of equations (24) and (27), equation (26) can be rewritten as

$$Nu = \frac{2(1+\Lambda\xi)^n}{\vartheta(\xi,1)-\vartheta_b(\xi)}. \tag{29}$$

Equation (29) proves that the local Nusselt number is a function of ξ, Λ and n. It is easily checked that for $n = 0$, the parameter Λ affects neither the dimensionless temperature ϑ nor the local Nusselt number. Indeed, the case $n = 0$ corresponds to uniform wall heat flux and, as is well known, in this case the local Nusselt number depends only on ξ [1].

FINITE DIFFERENCE SOLUTION OF THE BOUNDARY VALUE PROBLEM

In this section, a finite difference solution of the boundary value problem expressed by equation (22) is discussed.

The numerical solution of the boundary value problem (22) has been obtained by a finite difference method which employs a two-dimensional grid which is uniform with respect to r and non-uniform with respect to x. The linear system of equations obtained after the discretization has been solved by the successive overrelaxation method (SOR) [8]. The mesh has been obtained by the following method: the region $0 < x < L$ is subdivided in N_x intervals with a variable length given by

$$\Delta x_i = \Delta x_0 i^{0.5}, \quad i = 1,\ldots,N_x, \tag{30}$$

while the region $0 < r < r_0$ is subdivided in N_r intervals with a fixed length given by

$$\Delta r = \frac{r_0}{N_r}. \tag{31}$$

The quantity Δx_0 is determined by the normalization condition which requires that the sum of all the lengths Δx_i yields L. By this method, the mesh size in the x direction continuously decreases as $x = 0$ is approached. This structure of the mesh allows a higher precision in the numerical evaluation of the temperature field at the beginning of the thermal entrance region where steep axial temperature variations are expected.

The bulk temperature can be evaluated exactly by employing the integral energy balance equation [8]

$$\frac{dT_b}{dx} = \frac{2\alpha}{k\bar{u}r_0}q_w. \tag{32}$$

Since $q_w(x) = q_0(1+ax)^n$, equation (32) can be easily integrated and yields

$$T_b(x) = T_0 + \frac{2\alpha q_0}{k\bar{u}r_0 a}\frac{(1+ax)^{n+1}-1}{n+1} \tag{33}$$

if $n \neq -1$, and

$$T_b(x) = T_0 + \frac{2\alpha q_0}{k\bar{u}r_0 a}\ln(1+ax) \tag{34}$$

if $n = -1$. Equation (33) shows that, if $n < -1$, the bulk temperature tends to an asymptotic value for $x \to +\infty$ given by

$$T_{b\infty} = T_0 + \frac{2\alpha q_0}{k\bar{u}r_0 a(|n|-1)}. \tag{35}$$

The convergence of the overrelaxtion method has been obtained by evaluating the maximum value of the temperature change at any grid position in two subsequent iterations: the overrelaxation stops when this value is lower than 10^{-11} times the temperature difference $T_b(L)-T_0$.

A check on the global energy balance has been performed by comparing the exact value of $T_b(L)$, evaluated either by equation (33) or by equation (34), with the value of $T_b(L)$ obtained by a discrete sum approximation of the radial integral in equation (3), namely

$$T_b(L) \cong \frac{4\Delta r}{r_0^2}\sum_{j=1}^{N_r-1} T_{N_x,j}\left(1-\frac{r_j^2}{r_0^2}\right)r_j. \tag{36}$$

This comparison has shown that the relative errors in the energy balance due to the numerical approximation are less than 0.6%.

Table 1. Comparison of the numerical values of the local Nusselt number for $n = 0$ with those obtained analytically in ref. [9]

ξ	$N_r = 60$ $N_x = 300$	$N_r = 80$ $N_x = 400$	$N_r = 100$ $N_x = 500$	Cotta–Özisik [9]
0.000200	21.202	21.464	21.643	21.558
0.000300	18.367	18.708	18.833	18.790
0.000400	16.701	16.968	17.067	17.049
0.000500	15.509	15.733	15.821	15.813
0.000600	14.600	14.795	14.875	14.872
0.000700	13.873	14.049	14.121	14.123
0.000800	13.271	13.436	13.502	13.506
0.000900	12.767	12.919	12.980	12.985
0.001000	12.334	12.475	12.532	12.538
0.002000	9.8534	9.9442	9.9806	9.9863
0.003000	8.6714	8.7409	8.7687	8.7724
0.004000	7.9374	7.9950	8.0179	8.0200
0.005000	7.4232	7.4730	7.4927	7.4937
0.006000	7.0367	7.0811	7.0986	7.0986
0.007000	6.7327	6.7729	6.7887	6.7881
0.008000	6.4856	6.5226	6.5371	6.5359
0.009000	6.2798	6.3142	6.3277	6.3261
0.010000	6.1052	6.1375	6.1501	6.1481
0.020000	5.1712	5.1934	5.2019	5.1984
0.030000	4.7935	4.8123	4.8194	4.8157
0.040000	4.6009	4.6182	4.6248	4.6213
0.050000	4.4939	4.5106	4.5169	4.5139
0.060000	4.4321	4.4485	4.4547	4.4522
0.070000	4.3956	4.4120	4.4182	4.4162
0.080000	4.3738	4.3902	4.3965	4.3949
0.090000	4.3607	4.3772	4.3836	4.3823
0.100000	4.3527	4.3693	4.3758	4.3748
0.150000	4.3408	4.3580	4.3647	4.3645
0.200000	4.3389	4.3565	4.3635	4.3637

In Table 1, the numerical values of the local Nusselt number are reported as a function of ξ in the case of uniform wall heat flux ($n = 0$) and compared with those obtained by an analytical method in ref. [9]. The values obtained in ref. [9] are almost identical with those presented in ref. [1]. Three numerical solutions are reported in Table 1: one corresponding to $N_r = 60$ and $N_x = 300$, one corresponding to $N_r = 80$ and $N_x = 400$ and one obtained with $N_r = 100$ and $N_x = 500$. The comparison of these numerical results with the benchmark solution obtained in ref. [9] shows how the increase of the number of grid points determines the convergence to the exact solution. The numerical results reported in Table 1 are affected by a relative error which is higher at low values of ξ. This behaviour can be explained as follows. Since axial conduction in the fluid is neglected, the local Nusselt number tends to infinity in the limit $\xi \to 0$. Therefore, the temperature gradient in the neighbourhood of $\xi = 0$ is so steep that a finite difference solution loses its precision at very low values of ξ. This happens in spite of our choice of a mesh whose size continuously decreases as $\xi = 0$ is approached.

A comparison between the thermal entrance regions which correspond to $\Lambda = 1$ and $n = 0$, $n = 1$, $n = 2$ is presented in Fig. 2. This figure shows how the thermal entry length increases with n. The thermal entry length L_{th} is usually defined as the duct length required to achieve a value of local Nusselt number equal to γ times (with $\gamma > 1$) its fully developed value. The value of γ is arbitrary and the usual choice is $\gamma = 1.05$ [1]. The dimensionless thermal entry length is defined as $L^*_{th} = L_{th}/(2r_0Pe)$. Indeed, with $\gamma = 1.05$, the numerical evaluation of the local Nusselt numbers yields for $\Lambda = 1$ the following dimensionless thermal entry lengths: for $n = 0$, $L^*_{th} = 0.0433$; for $n = 1$, $L^*_{th} = 0.0466$; for $n = 2$, $L^*_{th} = 0.0506$. On the other hand, with $\gamma = 1.005$ one obtains for $\Lambda = 1$ the following thermal entry lengths: for $n = 0$, $L^*_{th} = 0.0882$; for $n = 1$, $L^*_{th} = 1.6185$; for $n = 2$, $L^*_{th} = 4.5869$. These results show how the thermal entrance region becomes considerably longer as n increases. Figures 3 and 4 illustrate the thermal entrance regions which correspond to $\Lambda = 1$ and $\Lambda = 100$, for $n = 1$ and $n = 2$, respectively. Indeed, these figures show that, for fixed n, if Λ is increased the thermal entrance region tends to become longer. Finally, Fig. 5 reveals that, for variations of n and Λ within the intervals $-2 < n < -1$ and $0.1 < \Lambda < 1$, the thermal entrance region does not sensibly change.

CONCLUSIONS

Forced laminar convection in a circular tube with a fully developed velocity profile and a prescribed axial distribution of wall heat flux has been considered. The effects of axial conduction and viscous dissipation in the fluid have been neglected. It has been proved that all the axial distributions of wall heat flux which fulfil equation (1) yield an asymptotic thermally developed regime. The asymptotic value reached by the Nusselt number is the same for all these axial distributions of wall heat flux and equals 48/11, i.e. the thermally developed value of the Nusselt number which occurs for axially uniform wall heat flux. Obviously, a uniform axial distribution of wall heat flux fulfils equation (1) as well as, for instance, all axial distributions which can be expressed as $q_w(x) = q_0(1+ax)^n$. It has been shown that the thermally developing local Nusselt number for wall heat fluxes given by $q_w(x) = q_0(1+ax)^n$ depends on the parameters $\xi = x/(2r_0Pe)$, $\Lambda = 2r_0Pe\,a$ and n. The thermally developing local Nusselt number for $q_w(x) = q_0(1+ax)^n$ has been determined by a finite difference method, for some values of n and Λ. Numerical results show that the local Nusselt number tends to 48/11 for $x \to +\infty$. Moreover, for positive values of n, it has been found that the thermal entry length increases considerably with n for a fixed value of Λ, and that the thermal entry length increases with Λ for a fixed value of n.

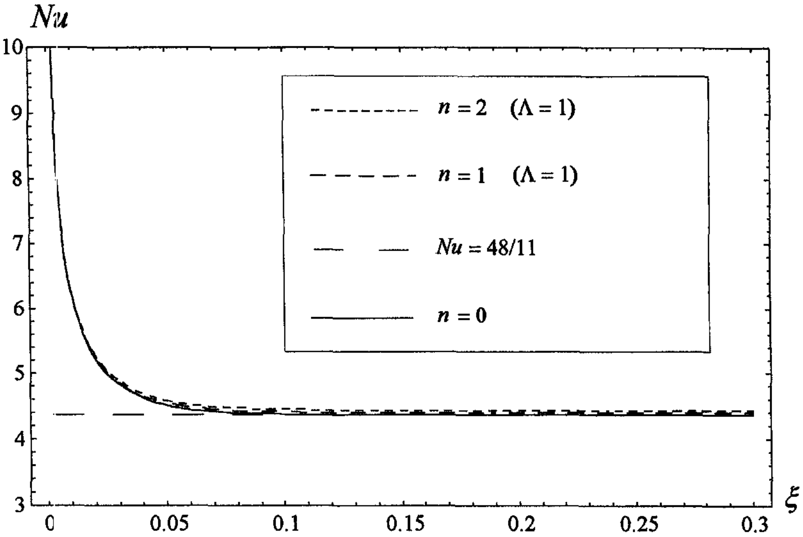

Fig. 2. Plots of Nu vs ξ for various values of n and for $\Lambda = 1$.

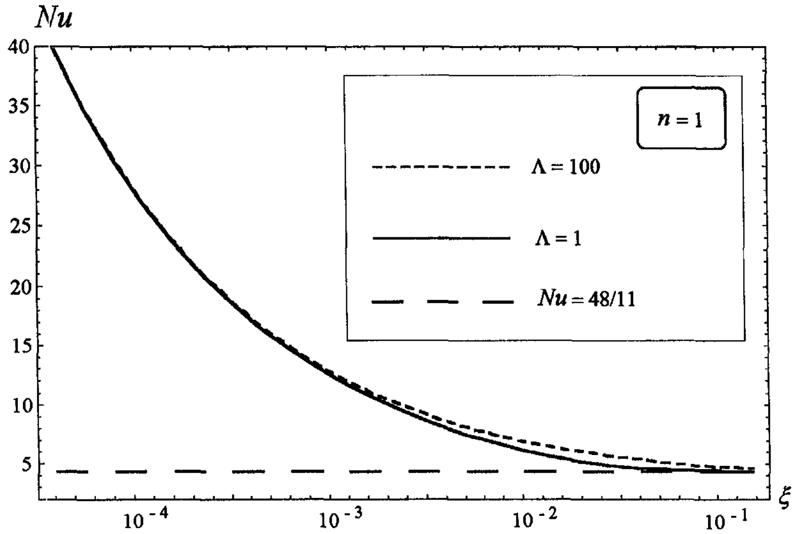

Fig. 3. Plots of Nu vs ξ (logarithmic scale) for $n = 1$ and for $\Lambda = 1$, $\Lambda = 100$.

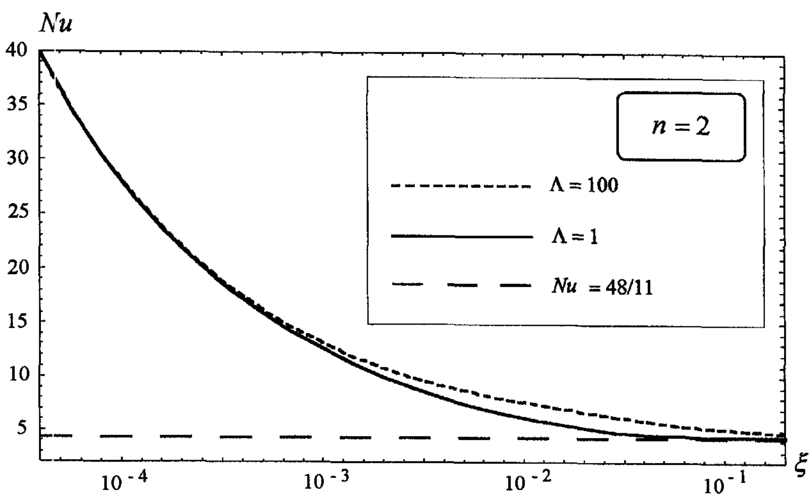

Fig. 4. Plots of Nu vs ξ (logarithmic scale) for $n = 2$ and for $\Lambda = 1$, $\Lambda = 100$.

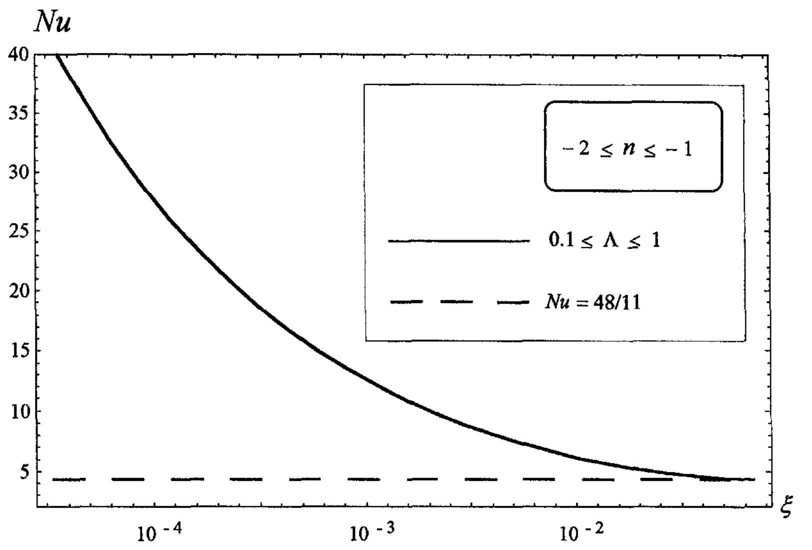

Fig. 5. Plot of Nu vs ξ (logarithmic scale) with $-2 \leqslant n \leqslant -1$ and $0.1 \leqslant \Lambda \leqslant 1$.

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